

# Mastery in Mathematics

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# MASTERY IN MATHEMATICS



What does it mean to understand in mathematics?

Consider someone you know that you would identify as having mastery of mathematics, why do you choose this person?

What is it that this person can do that makes them 'mathematical' or seem good at mathematics?

# Mastery in Mathematics



- A deep conceptual understanding
- Ability to understand and use mathematical vocabulary
- An ability to spot patterns, reason and generalise
- Flexibility and resilience when approaching problems
- Fluency and efficiency
- An ability to solve problems in a range of contexts
- *Requires a particular ethos and identity*
- *Requires opportunities to overlearn*
- *Requires collaboration*
- *Places emphasis on discussion and proof*
- *Pupils move at broadly the same pace*

# Assumptions/discourses about ability in mathematics

“You can either do maths or you can’t”

“Only some people are good at maths”

“You are either mathematical/scientific or arty/creative”

“I was never any good at maths” (parent to child)

“When will I ever use this in everyday life?”

Can you think of any others?

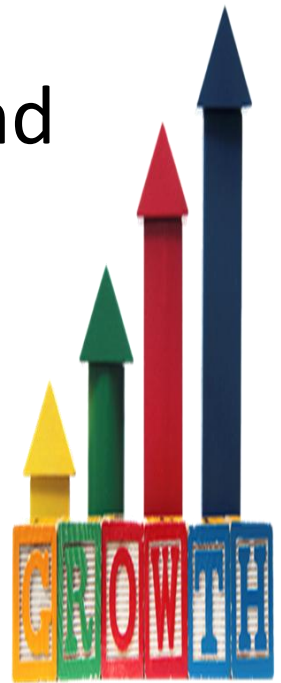


## **Growth mindset (v. fixed mindset)**

“Do people with this mindset believe that anyone can be anything, that anyone with proper motivation or education can become Einstein or Beethoven? No, but they believe that a person’s true potential is unknown (and unknowable); that it’s impossible to foresee what can be accomplished with years of passion, toil, and training.”

(Dweck, 2006)

We need to allow ‘different rates of the developmental trajectory’ (Stigler, 2011)



## Charlie Stripp's (Head of NCETM) Blog

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“ *I think it may well be the case that one of the most common ways we use differentiation in primary school mathematics... has had, and continues to have, a very negative effect on the mathematical attainment of our children at primary school and throughout their education.* ”

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- Labels and ideas of smartness and giftedness carry with them fixed ideas about ability, suggesting to students that they are born with a gift or a special brain. When students are led to believe they are gifted, or they have a “math brain” or they are smart and later struggle, that struggle is absolutely devastating. Students who grow up thinking that they have a special brain often drop out of STEM subjects when they struggle.

- Jo Boaler (2017)

# Liping Ma (1999)

$$1 \frac{3}{4} \div \frac{1}{2}$$

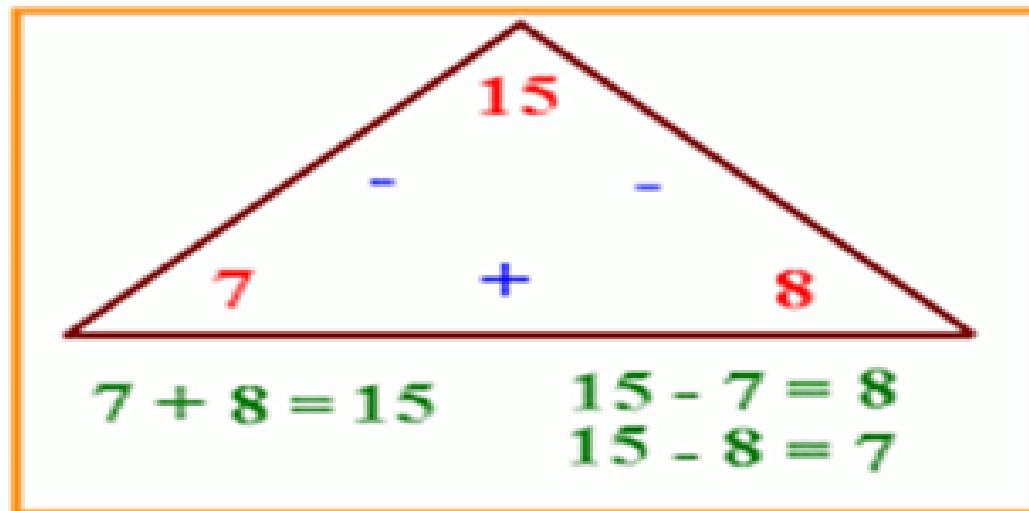


***Knowing and Teaching Elementary Mathematics: Teacher's Understanding of Fundamental Mathematics in China and the United States***



What is meant by mastery and depth?

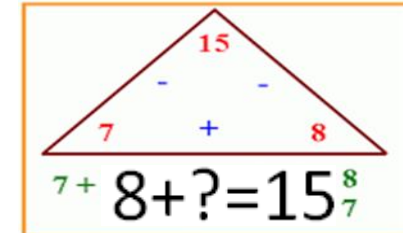
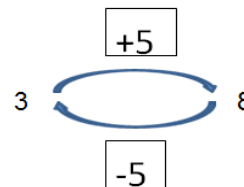
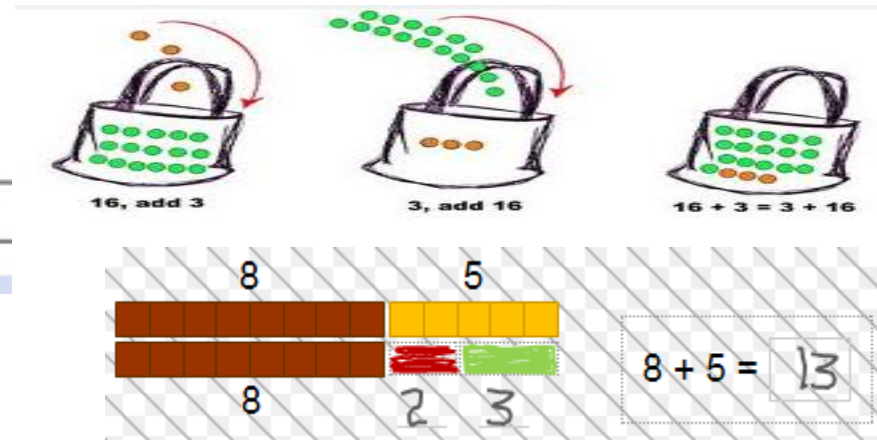
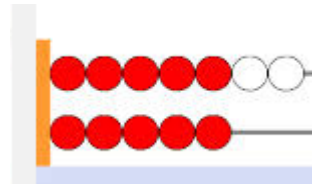
*Consider what you would expect a child to be able to do if they had a deep understanding of number facts?*

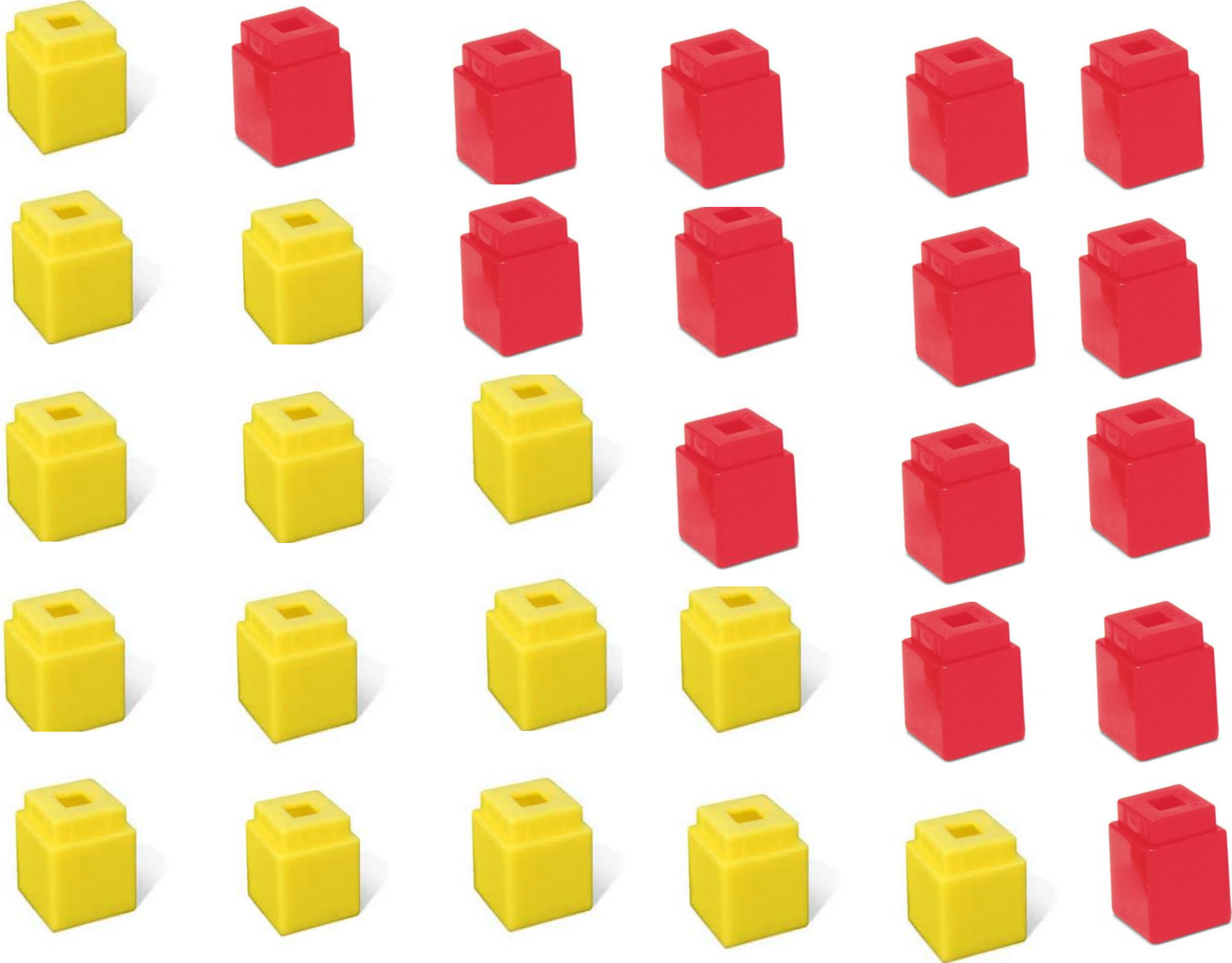


**What is 'mastery' of number bonds?**

# Mastery of Number Bonds

- **Memorising facts**
- **Using facts**
  - Deriving new facts
  - Bridge
- **Laws and principles**
  - Inverse relationships
  - Commutativity
  - Associativity
- **Patterns and connections**
  - Modelling with apparatus
  - Comparing apparatus
  - Explanation
  - Same different/ what do you notice
  - Own examples
- **Generalising**  
*'How many ways can you make 7?'*





Whole	Part	Part
5	5	0
5	4	1
5	3	2
5	2	3
5	1	4
5	0	5

$$\square + 3 = 5$$

$$5 = 3 + \square$$

$$\square - 2 = 3$$

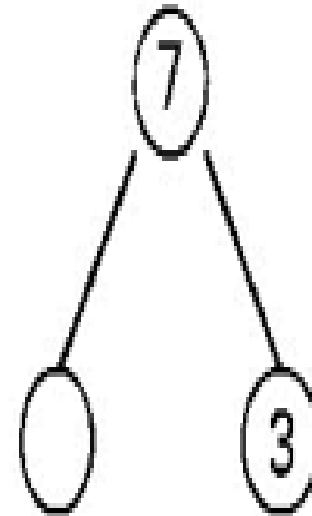
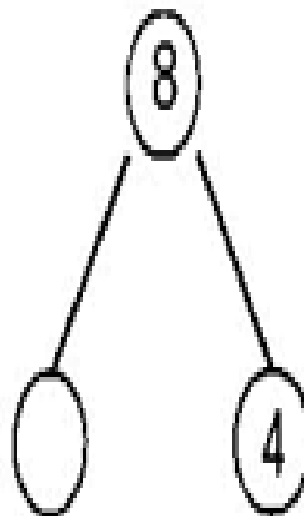
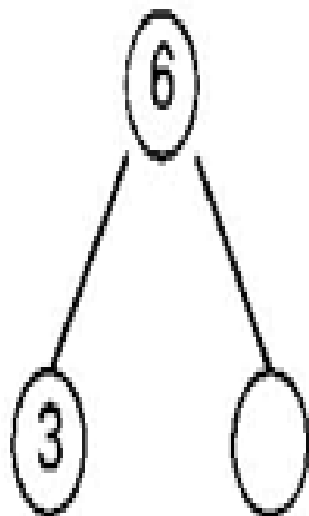
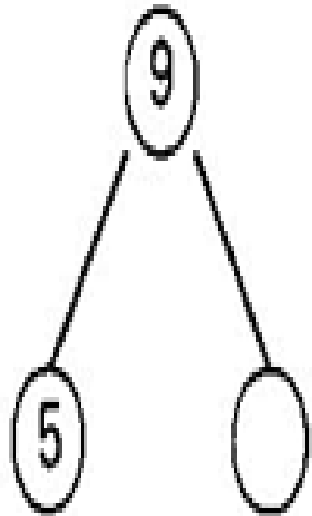
$$\square - 3 = 2$$

$$2 + \square = 5$$

$$3 = 5 - \square$$

# Learning facts as a trio

Teachers can use the triad format in simple written exercises, such as 'fill in the missing numbers'.



$$9 - ? = 4$$

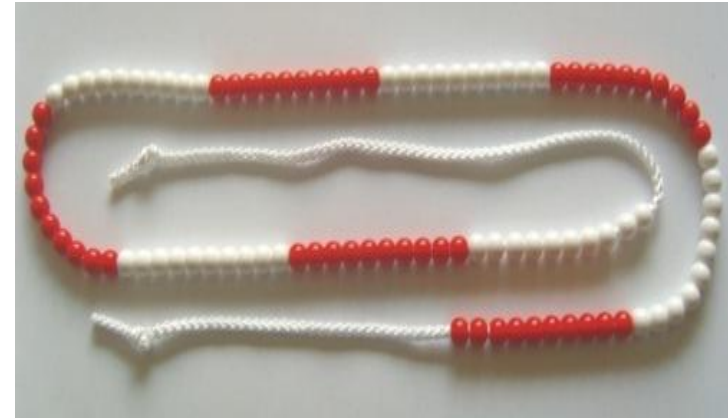
$$5 + ? = 9$$

$$? - 5 = 4$$

$$9 = 4 + ? \dots$$

# Patterns, relationships and structures

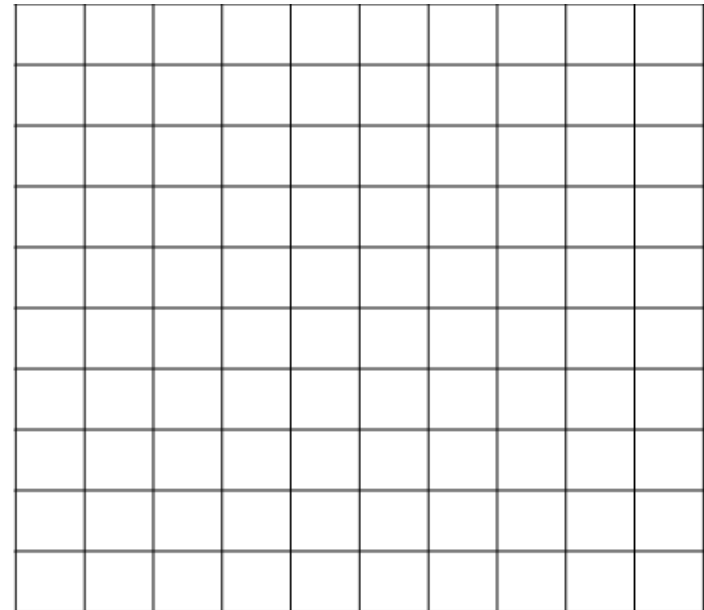
- $2+3$ ,  $12+3$ ,  $22+3$ ,  $32+3$
- $6+?=10$ ,  $16+?=20$ ,  $26+?=30$
- $10-1$ ,  $20-1$ ,  $30-1$



What do you notice?

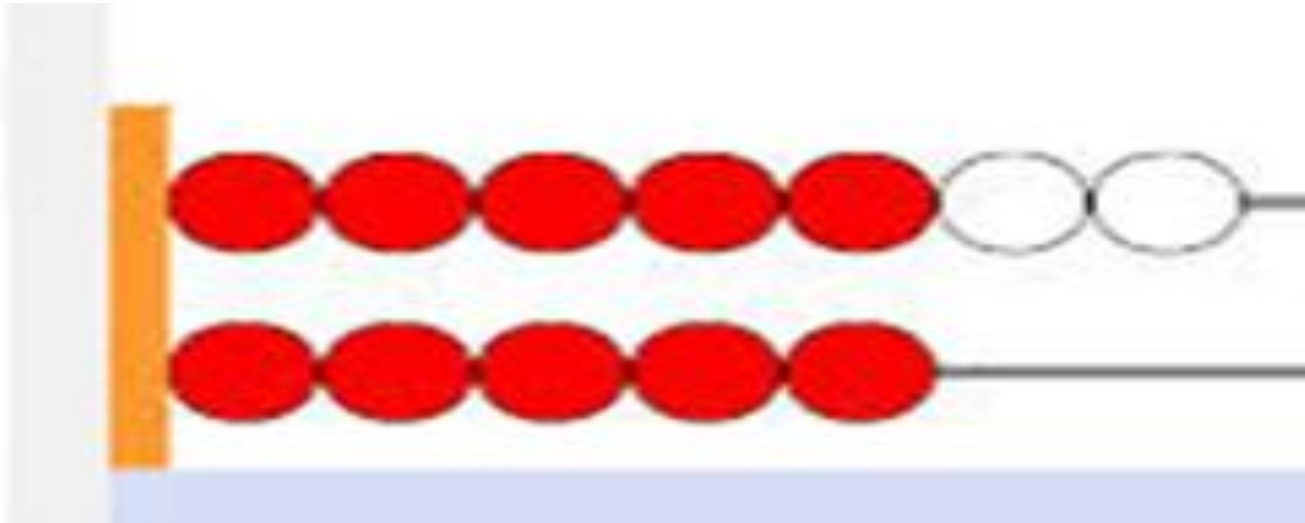
Will it always happen?

Why does it keep happening?



# Patterns, relationships and structures

If you know  $5+5=10$



What else do you know?

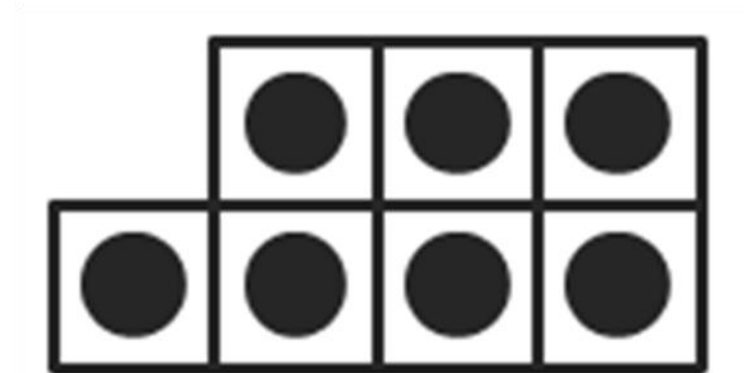
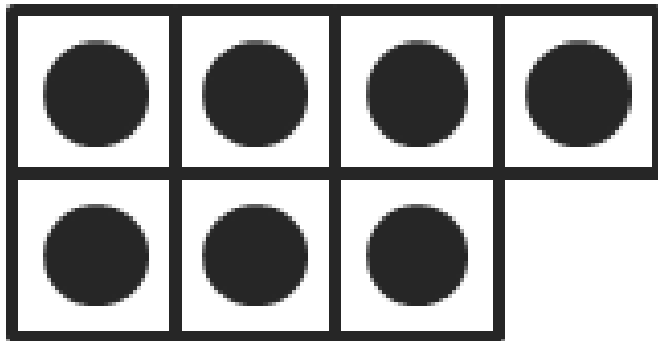


**Bridging through multiples of 10**



# Visualising a structure or a relationship

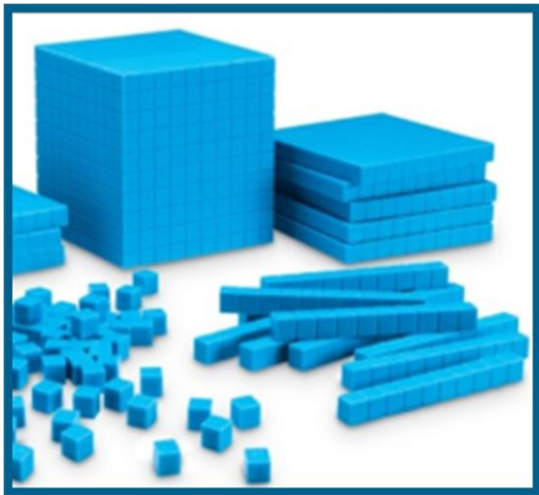
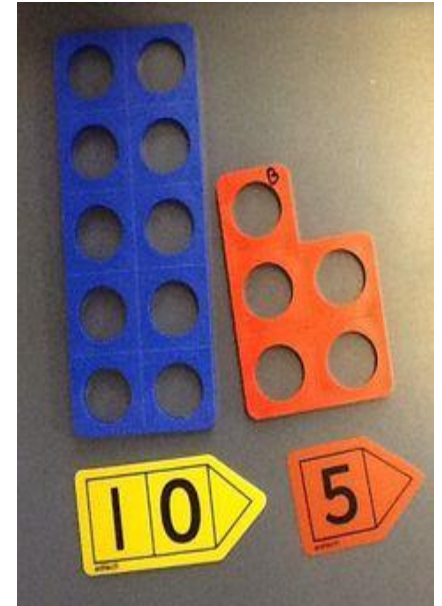
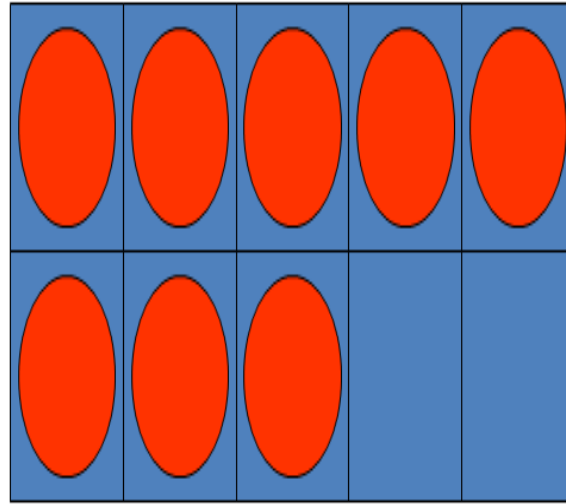
What happens when we add two odd numbers?



# Resources and Structures



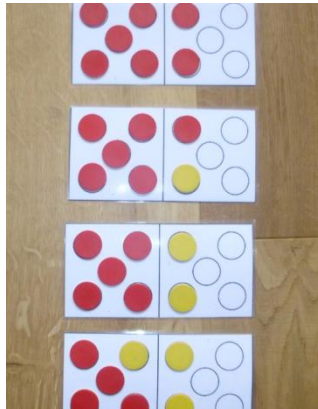
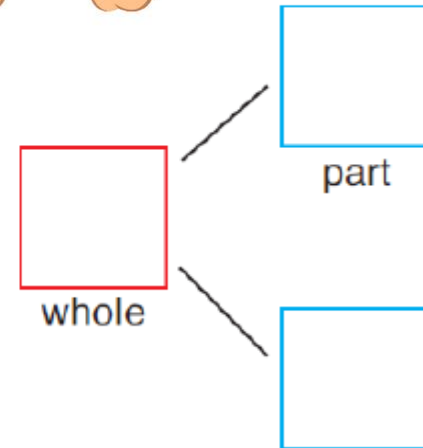
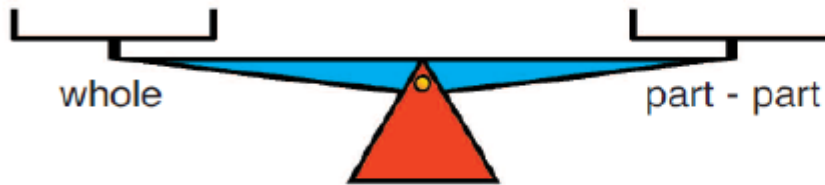
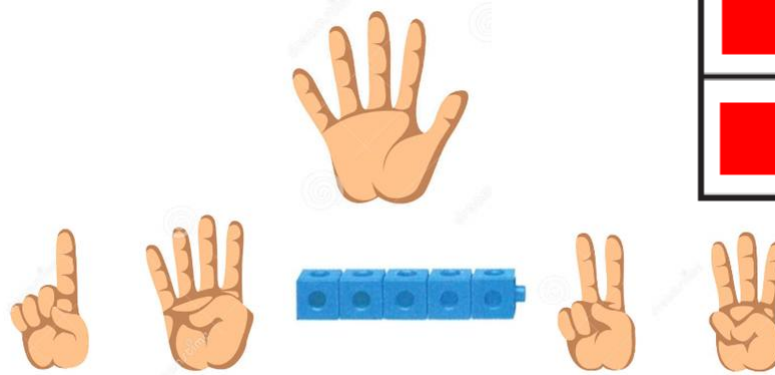
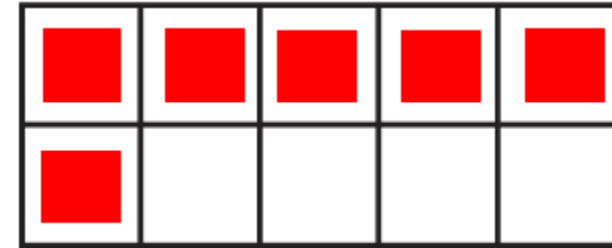
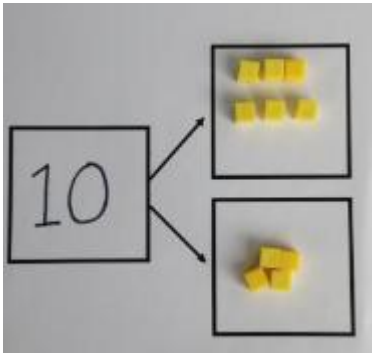
# Link resources, contexts and models



Here  $13 \times 5 = (10 \times 5) + (3 \times 5)$ .



# Some more models...

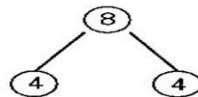


$$10 = 6 + 4$$

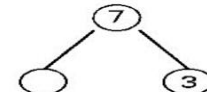
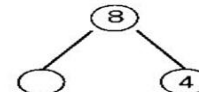
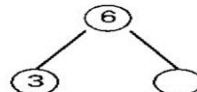
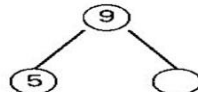
$$10 - 6 = 4$$

$$10 - 4 = 6$$

$$10 = 4 + 6$$

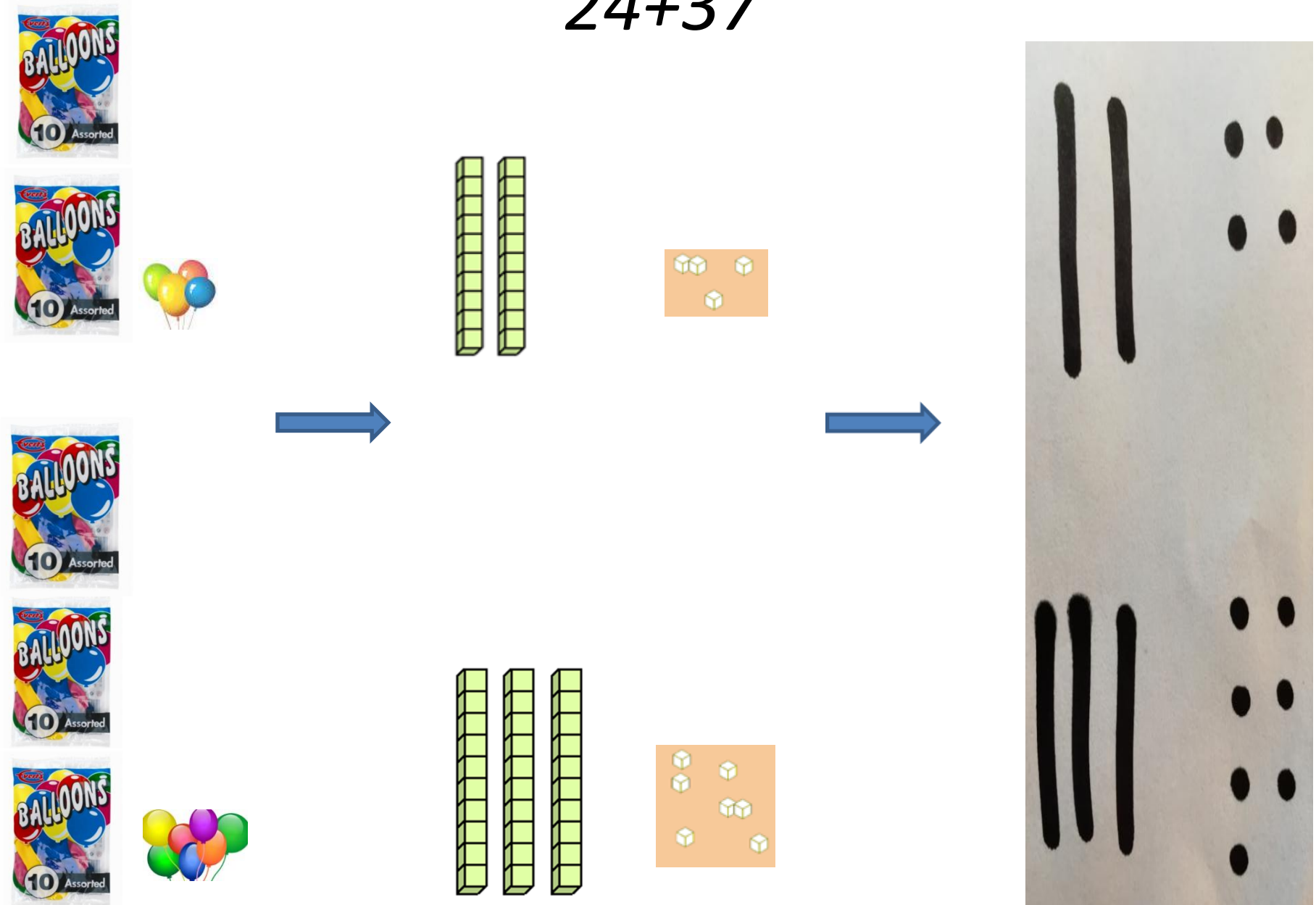


Teachers can use the triad format in simple written exercises, such as 'fill in the missing numbers'.



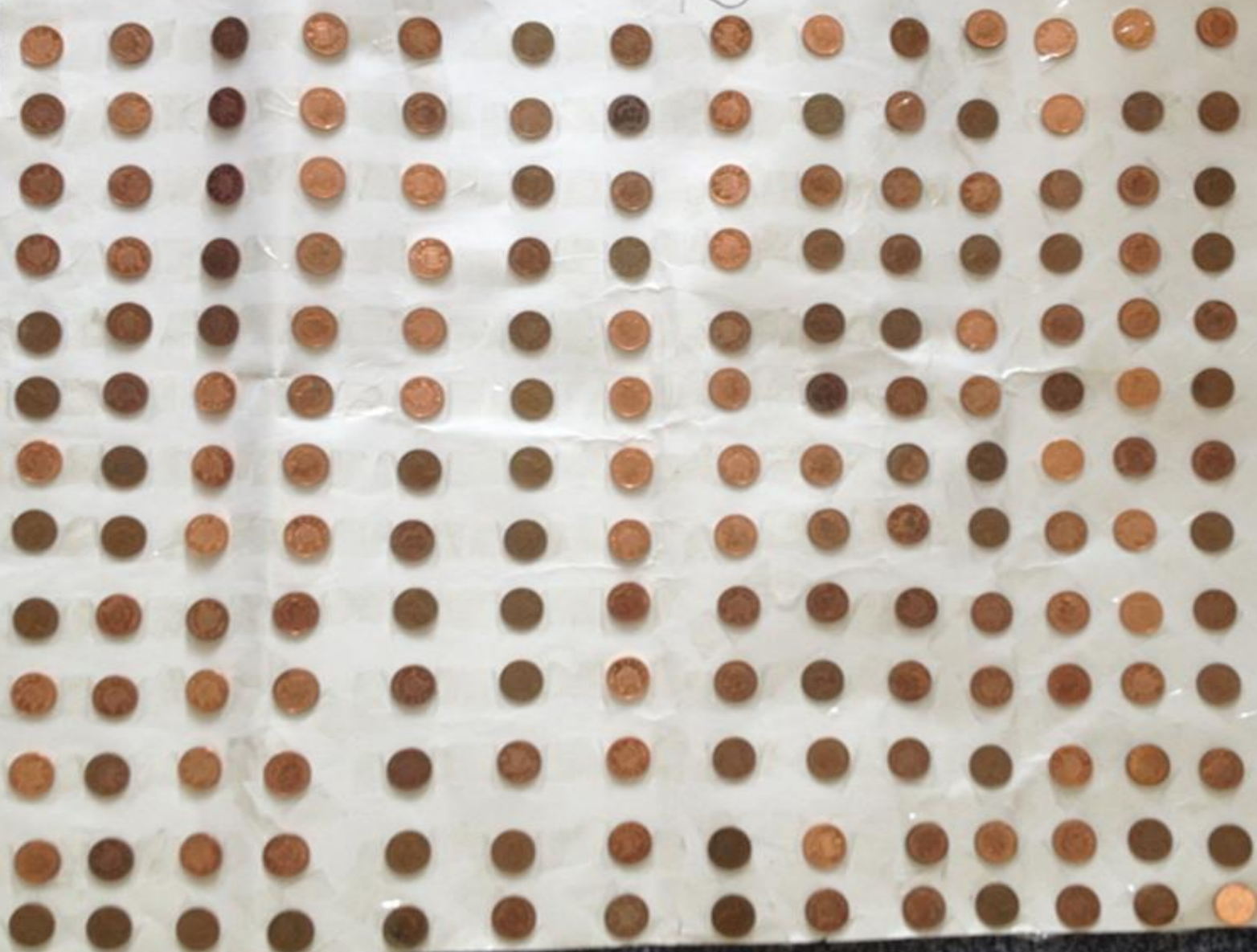
# Models, Language, Recording, Visualising

## $24+37$

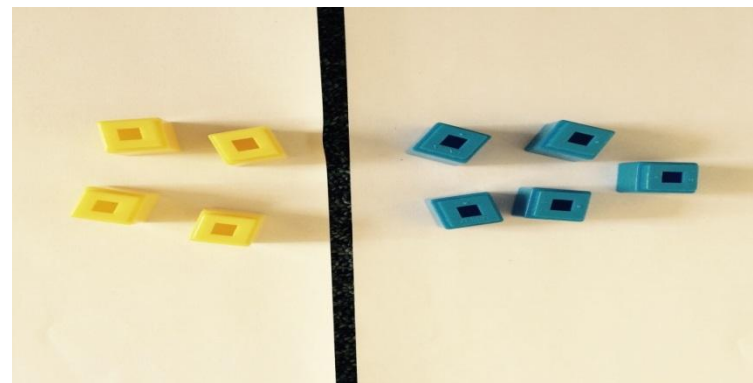




How Many 1ps?



## Developing Explanation



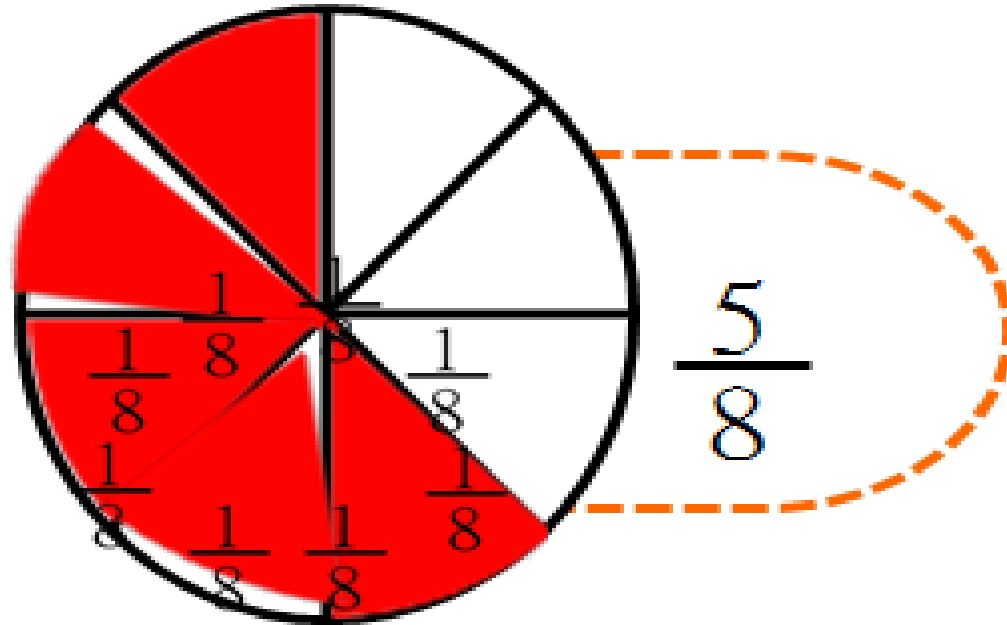
One, two, three, four, five. There are five cubes here.

One, two, three, four. There are four cubes here.

One, two, three, four, five, six, seven, eight, nine. There are nine cubes altogether.

Five plus four is equal to nine.

**Write down the fractions to show the red parts as a fraction of the circle.**



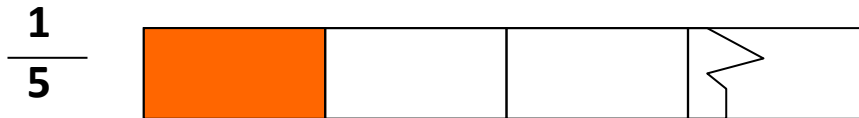
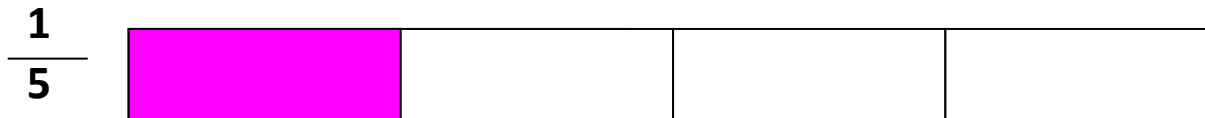
The whole is divided into \_\_\_\_ equal parts. Each part is worth \_\_\_\_ of the whole. If I take \_\_\_\_ of the parts then this is \_\_\_\_ times \_\_\_\_ of the whole. This is \_\_\_\_.



## Reasoning NOT Rushing

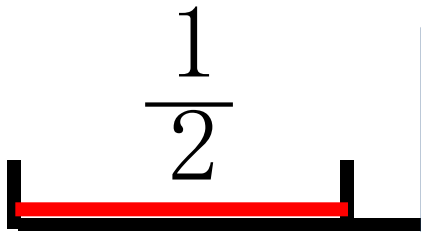
**2 paper tapes were broken, can you guess which original paper tape is longer?**

**Why? How do you get your answer?**

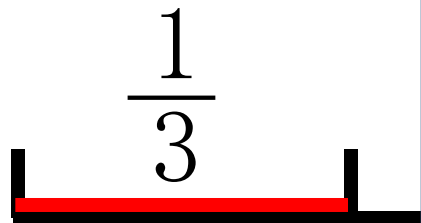


# Think: Which line is longer?

First:



Second:



How might you calculate?

$$4 \times 376 = 2 \times ?$$

Providing a model to explain or prove

True or false

$$3 \times 40 = 30 \times 4$$

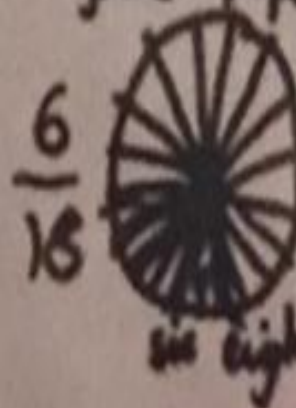
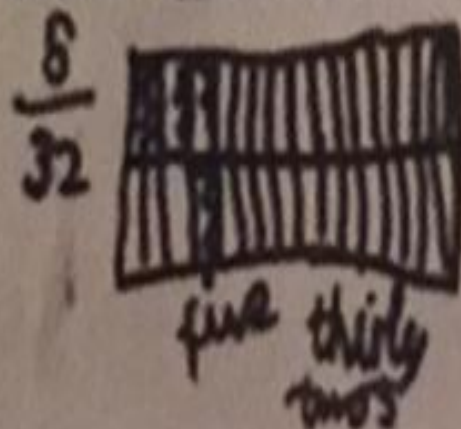
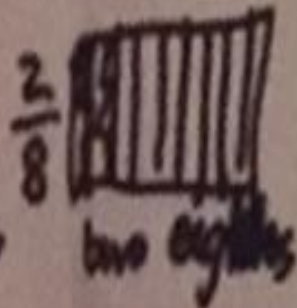
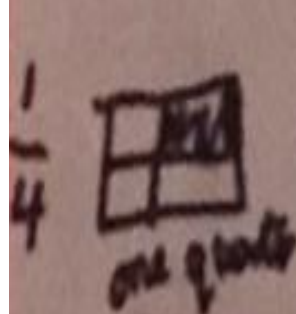
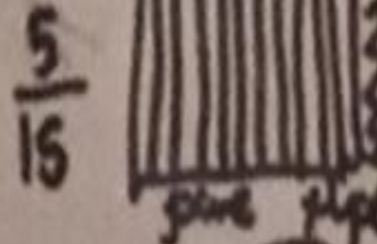
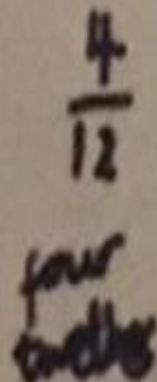
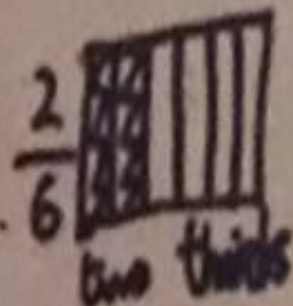
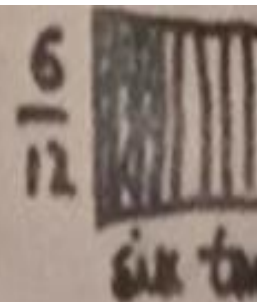
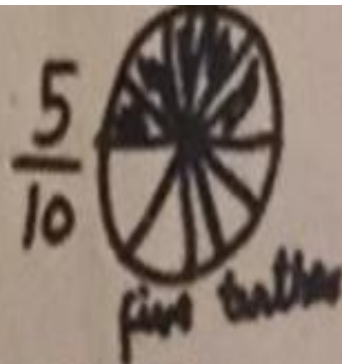
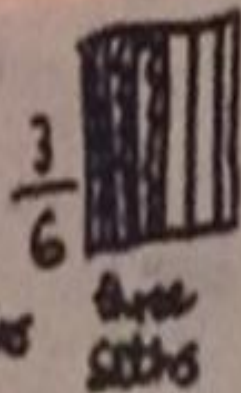
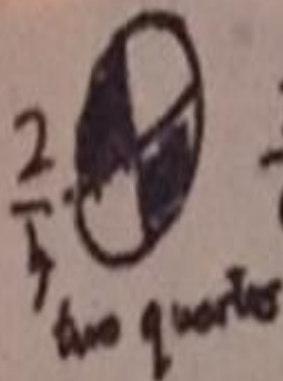
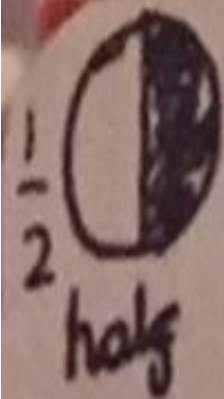
Prove!

One table using dienes

one table using squared paper to make arrays

one group using place value counters

Will it always work?



# Generalising about Equivalence

Oscar's generalisation:

even  $\rightarrow \frac{2,500,000}{5,000,000}$

"You can only use fractions with an even denominator when you are making fractions which are equivalent to a half."

$$\frac{6}{12}$$

← numerator  
← denominator

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

$$\frac{1}{3} = \frac{2}{6} = \frac{4}{12} = \frac{3}{9} = \frac{5}{15}$$

Marc's generalisation:

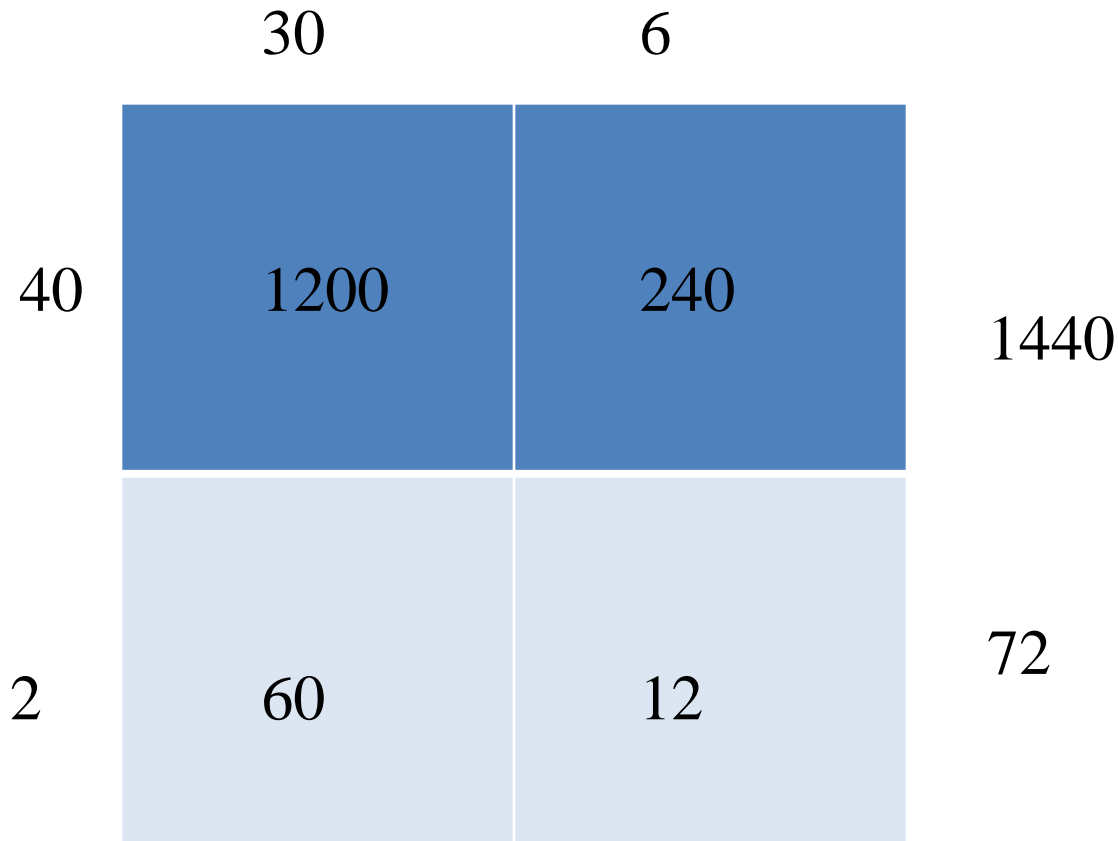
"The denominator has to be in the three times table when you are making fractions equivalent to a third."

# Can you make a generalised statement?

- Look at  $7 \times 3 = 21$  and  $9 \times 3 = 27$
- One of the factors has increased by 2, what happens to the product?
- Consider another example:
- $7 \times 3 = 21$  and  $7 \times 5 = 35$
- One of the factors has increased by 2, what happens to the product?
- Make up an example of your own
- *Make a generalised statement about what happens when you increase one of the factors by 2. Prove it!*

# 36x42

*What is the same and what is different?*



$$1440 + 72 = 1512$$

		3	6
X		4	2
		7	2
1	4	4	0
1	5	1	2



# What is the same and what is different?

- I have 100 cakes. One box holds 8 cakes.  
How many boxes do I need to hold all the cakes?
- I have £100. Tickets to the film cost £8.  
How many people can I take?
- After a meal in a restaurant the bill comes to £100. The bill is shared equally between 8 people, how much do they each pay?

Reason why about the mathematics

The answer is only the beginning

Using

< > or =

Fill in •

- $4 \times 6 \bullet 10$

- $24 + 4 \bullet 4$

- $24 \bullet 6 + 4$

- $3 \times 4 \bullet 6 \times 2$

- $9 \bullet 4 + 5$

- $4 \times 1 \bullet 5$

# Rich Tasks Rich Questions NOT Rushing

I have 36 chocolates, how many different rectangular boxes can you suggest to the manufacturer?



# Multiple Models: Variation



Canada's capital is Ottawa



Kenya's capital is Nairobi



Switzerland's capital is Bern



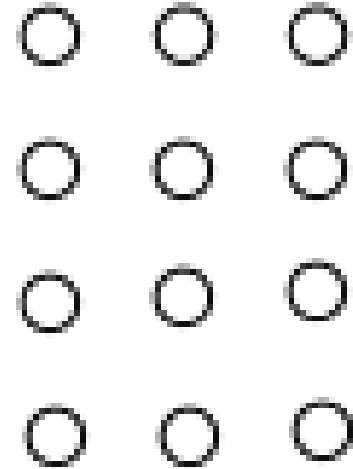
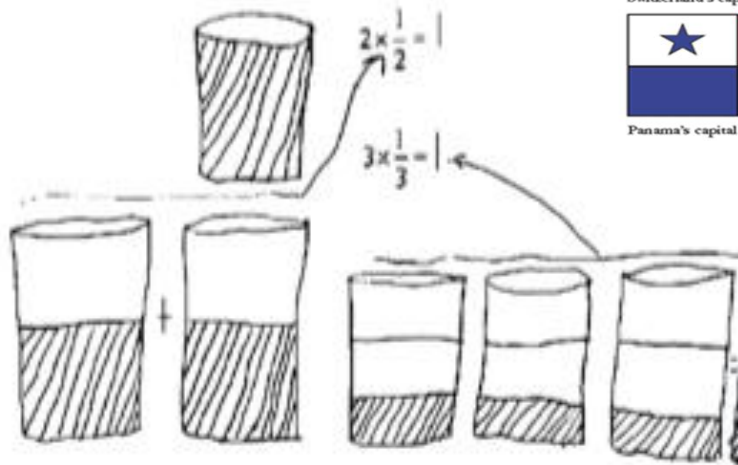
Wales's capital is Cardiff



Panama's capital is Panama City



Australia's capital is Canberra



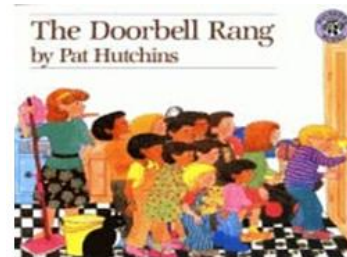
# Fractions

*‘No area of elementary mathematics is as mathematically rich, cognitively complicated, and difficult to teach as fractions, ratio and proportionality’ Smith (2002)*



# Equal Sharing and fractions as an extension of sharing with whole numbers

- 6 biscuits between 2 then 3 children etc
- $3/5/7$  biscuits between 2 children
- 5 biscuits between 4 children
- 4 biscuits between 3 children
- 5 biscuits between 3 children
- 7 biscuits between 4 children



# Equal Sharing



## **4 cookies 3 kids**

- Joe: How would you share them?
- Leah: One, one, one and there is one left. Then they each get one third, one third, one third
- Joe: So how much does each kid get?
- Leah: They get one whole one and one third

## **5 cookies and 3 kids**

- Leah: One, one, one and there is two more left. They get a third, a third, a third and then a third, a third, a third
- Joe: So how much do they each get?
- Leah: They get one and two thirds

## **7 cookies shared between 4 kids**

*Leah:* That's a hard one, maybe I can't do it.

*Joe:* Think about what you did to solve the other two.

*Leah:* Whole, whole, whole, whole, then there's three more left. Um, three more cookies left. Then you break up one into halves, then there are two left. And, another into half, half. Break the last one into quarter, quarter, quarter, quarter.

*Joe:* Great! How much does each kid get?

*Leah:* One whole, one half and one quarter.

Leah is four.

(Anthony and Walshaw, 2007)

## **Contrast this with my daughter's homework in year 2...**



# What can we do as parents

- Be a maths enthusiast
- Get them to talk about the maths they have done each day
- Get them to explain the maths to you
- Support them through practice with memory aspects of maths:
  - Vocabulary
  - Facts
  - Procedures
- Encourage them to make connections
  - between different aspects of maths
  - real life maths
- Go shopping with real money and play games

# Any Questions?

